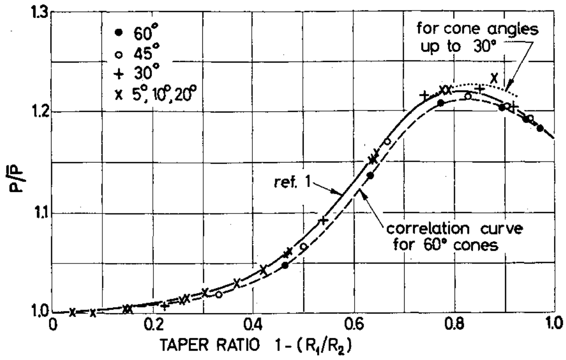

Fig. 1 Correlation between critical pressure of conical and equivalent cylindrical shells with re-emphasis on cone angle dependence (from Fig. 2 of Ref. 1).

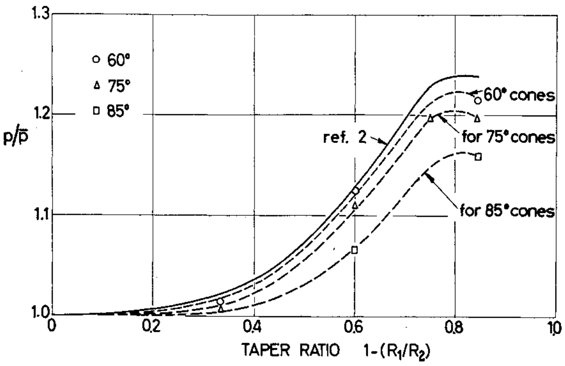

Fig. 2 Correlation curves for conical shells with large cone angles.

2 were based on typical shells of fairly high ratios of small radius to thickness (R_1/h), the range being 250–2000. If one now computes the ($p/\bar{p}$) ratio for a typical shell and then repeats the calculations for the same shell but with different thicknesses, a decrease in ($p/\bar{p}$) with increasing thickness is noted. This decrease is very small for large (a/h) ratios, where (a/h) is an alternative thickness ratio criterion, a being the distance along a generator of the small end of the conical frustrum from the vertex; but it becomes appreciable for thicker shells of (a/h) ratios below 300. For example, although for a typical shell a decrease of only 1.5% in ($p/\bar{p}$) was found when the (a/h) ratio was changed from 700 to 300, a 6% decrease resulted when the (a/h) ratio was changed from 300 to 50.

Since the calculations of Ref. 1 did not go below (a/h) = 290, it is not surprising that the decrease of ($p/\bar{p}$) with (a/h), or rather with (R_1/h), was found to be very small and was hence obliterated by averaging out. But, if one intends to apply the correlation curves of Ref. 1 or 2 to shells with (a/h) below 250, this effect may be significant.

It may be pointed out that both cone angle dependence and thickness ratio effects of the same order were found when a similar correlation with equivalent cylindrical shells was carried out for orthotropic shells.[3]

References

[1] Seide, P., "On the buckling of truncated conical shells under uniform hydrostatic pressure," *Proceedings of the IUTAM Symposium on the Theory of Thin Elastic Shells, Delft, Holland, August 1959* (North-Holland Publishing Company, Amsterdam, 1960), p. 363.

[2] Singer, J., "Buckling of conical shells under axisymmetrical external pressure," J. Mech. Eng. Sci. **3**, 330–339 (1961).

[3] Singer, J. and Fersht-Scher, R., "Buckling of orthotropic conical shells under external pressure," Technion Res. and Dev. Foundation, Haifa, Israel, TAE Rept. 22 (September 1962).

Addendum: Dual Electric-Nuclear Engine

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In a recent paper,[1] comparison was made between all-nuclear rockets and dual electric-nuclear rockets for Mars trips. It was found that the dual electric-nuclear system reduced the gross weight of the vehicle in initial earth orbit to 0.4–0.6 of the gross weight of the all-nuclear vehicle. Comparison was not made, however, with an all-electric vehicle.

A recent parametric study by Moeckel[2] indicates that for a fast round trip comparable to that in Ref. 1. (347 days), and for a power plant specific weight equal to that for the present typical vehicle (8.3 lb/kw), all-electric engines will have about the same gross weight as an all-nuclear vehicle for equal payloads returned to earth. Although details of the mission profile in Moeckel's study are somewhat different, it is believed that the over-all comparisons in the two studies are consistent.

It is therefore possible to conclude that the dual electric-nuclear rocket system would reduce the gross weight of a comparable all-electric vehicle to approximately the same degree (~0.4–0.6) that it would reduce the gross weight of an all-nuclear vehicle.

References

[1] Levoy, M., "Dual electric-nuclear engine," AIAA J. 1, 1298–1302 (1963).

[2] Moeckel, W. E. "Electric propulsion systems for Mars missions," Am. Astronaut. Soc. Symposium on the Exploration of Mars, Preprint No. 4 (June 1963).

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Comment on "Velocity Defect Law for a Transpired Turbulent Boundary Layer"

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The interesting note by Mickley and Smith[1] suggests that the hypothesis advanced by Clauser[2] for the turbulent boundary layer can be extended to the problem of the transpired boundary layer by a substitution of the friction velocity U_τ^* based on the maximum shear stress. It has been established that for the simple case of zero axial pressure gradient and for an impermeable plate, U_τ^* is a maximum at the wall. Where a disturbance exists at the wall due to a pressure gradient or indeed mass transport, a single solution of the velocity distribution function is no longer applicable. However, in the outer region of the boundary layer, the momentum equation for the flow is reduced to the Reynolds stress equations:

$$\bar{u}\frac{\partial \bar{u}}{\partial y} + \bar{v}\frac{\partial \bar{u}}{\partial x} = \epsilon\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right) \qquad \text{etc.} \tag{1}$$

This, in essence, suggests that the eddy diffusivity ϵ of the

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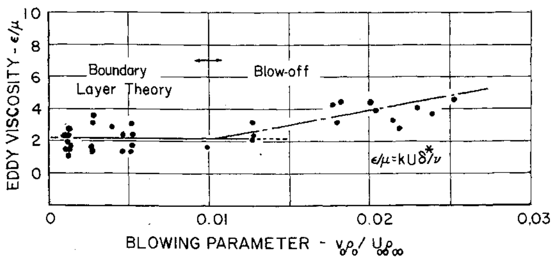

Fig. 1 Test of the constancy of the eddy viscosity in the ratio portion of a turbulent boundary layer over a flat plate with uniform blowing. (Data taken for air-air system.[3])

outer region is independent of pressure gradient or perturbing influence at the wall. It was pointed out earlier by this author[3] that if this was indeed the situation, as in the case of a pressure gradient, the eddy diffusivity of the outer region could also be treated as a constant with mass addition at the wall, provided the momentum of injection normal to the plate did not exceed the momentum of the shear layer. Thus, using the Clauser expression for the eddy viscosity

$$\epsilon = k \cdot \bar{U}_\infty \cdot \delta^* \rho_\infty \tag{2}$$

where k is a function of δ^*/τ_0 and the injection parameter $V_0\rho_0/U_\infty\rho_\infty$, one can investigate various equilibrium velocity profies with uniform wall injection to determine the behavior of ϵ in the outer portion of the layer.

The results of such a calculation for air injected uniformly along the plate and normal to the main air flow is shown in Fig. 1 for an assumed constant $k = 0.018$. It is clear that up to the so-called "blow-off" point, the assumptions maintained by Clauser for eddy viscosity of the Reynolds stress layer are still correct, and it appears that we are justified in considering k as a constant in calculating ϵ in the region before separation. The variation of ϵ may be attributed at this stage to experimental inaccuracies in the experimental work.

The main problem appears to be centered on the nature of the wall shear layer under the influence of mass addition. The fact is that only for injection ratios of less than the separation value can the profile be described by the Clauser model. As yet there is no process method by which the shear velocity $U_\tau{}^*$ can be related to the mass addition parameter, $V_0\rho_0/U_\infty\rho_\infty$. However, it appears from the data of Fig. 1 and Ref. 3, provided the mass addition ratio is less than 0.01, that the outer shear layer is unaffected by conditions at the wall. The observation by the authors is essentially correct, but one must wait upon results of additional measurements made close to the wall for a broader range of injection parameters before accepting this model.

References

[1] Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. **1**, 1685 (1963).

[2] Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press, Inc., New York, 1956), Vol. 4, pp. 1–51.

[3] Hacker, D. S., "Interferometric investigation of the stability of a turbulent boundary layer with mass addition," Am. Soc. Mech. Engrs. Paper 58A-249 (1958).

Reply by Authors to D. S. Hacker

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The authors appreciate the interest of Hacker in their earlier paper.[1] The purpose of that paper was to present evidence that when the friction velocity is based upon the maximum shear, the outer portion of a transpired turbulent boundary layer may be described by a velocity defect law similar to that which describes nontranspired boundary layers. Additional evidence is certainly desirable and the authors are currently carrying out additional experimental measurements.

The maximum shear appears to provide a more suitable boundary condition for the outer flow than the wall shear. If, as the experimental data indicate, the eddy viscosity of the outer flow is independent of y, an analysis along the lines of Clauser[2] as suggested by Hacker[3] is possible. The authors have embarked on such calculations.

The problem of treating the inner flow and of connecting it with the outer flow remains to be solved.

References

[1] Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. **1**, 1685 (1963).

[2] Clauser, F. H., "The turbulent boundary layer," *Advances in Applied Mechanics* (Academic Press, Inc., New York, 1956), Vol. 4, pp. 1–51.

[3] Hacker, D. S., "Interferometric investigation of the stability of a turbulent boundary layer with mass addition," Am. Soc. Mech. Engrs. Paper 58-A-249 (1958).

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